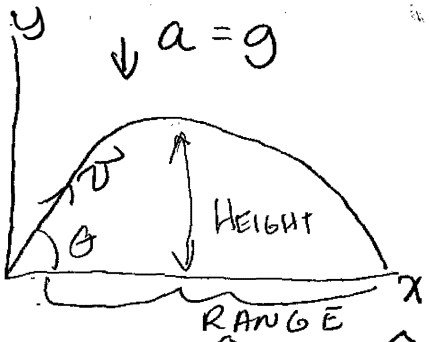
Projectile Motion - Two dimensional motion on the Eath

EQUATIONS:

$a = \frac{v - v_0}{t}$; $\frac{v + v_0}{2} = \frac{r - r_0}{t}$; $s = \frac{at^2}{2} + v_0 t$

NOW IN OUR GRAVITY FIELD WE SEE AN ARC:



$v_{0x} = v_0 \cos\theta$

$v_{0y} = v_0 \sin\theta$

THIS DEMONSTRATION SHOWS WHAT GOES UP, MUST GOES DOWN.

$a = a_x \hat{i} + a_y \hat{j}$; $a_x = 0 m/s$; $a_y = -g$ (ACCELERATION DUE TO GRAVITY) $9.81 m/s^2$

$\boxed{a_x = 0 m/s^2}$

$a_x = \frac{v_x - v_{0x}}{t}$

$0 m/s^2 = \frac{v_x - v_{0x}}{t}$

$0 = v_x - v_{0x}$

$+v_{0x} \quad +v_{0x}$

$\boxed{v_x = v_{0x}}$

$\frac{v + v_0}{2} = \frac{r - r_0}{t}$

$\frac{v_{0x} + v_{0x}}{2} = \frac{x - x_0}{t}$

$\boxed{a_y = -g}$

$a_y = \frac{v_y - v_{y0}}{t}$

$-gt = \frac{v_y - v_{y0}}{t}$

$-gt = v_y - v_{y0}$

$+v_{y0} \quad +v_{y0}$

$\boxed{v_y = v_{y0} - gt}$

$\frac{v + v_0}{2} = \frac{r - r_0}{t}$

$\frac{v_{y0} - gt + v_{y0}}{2} = \frac{y - y_0}{t}$

$t\,v_{0x} = x - x_0$

$\frac{t}{t}$

$v_{0x}t = x - x_0$

$+x_0 \quad +x_0$

$$\boxed{x = v_{0x}t + x_0}$$

$t\left(\frac{v_{y0} - gt + v_{y0}}{2}\right) = \left(\frac{y - y_0}{t}\right)t$

$v_{0y}t - \frac{gt^2}{2} = y - y_0$

$+y_0 \quad +y_0$

$$\boxed{-\frac{gt^2}{2} + v_{0y}t + y_0 = y}$$

NOW WE WANT TO FIND HEIGHT ($v_y = 0\,m/s$)

$v_y = v_{y0} - gt$

$-v_{y0} \quad -v_{y0}$

$-v_{y0} = -gt$

$+v_{y0} \quad +v_{y0}$

$v_{y0} = gt$

$\frac{v_0 \sin\theta}{g} = \frac{gt}{g}$

$$\boxed{\frac{v_0 \sin\theta}{g} = t_{HEIGHT}}$$

$-\frac{gt^2}{2} + v_{0y}t + y_0 = y$

$-\frac{g}{2}\left(\frac{v_0 \sin\theta}{g}\right)^2 + v_0 \sin\theta\left(\frac{v_0 \sin\theta}{g}\right) + y_0 = y$

$-\frac{v_0^2 \sin^2\theta}{2g} + \frac{v_0^2 \sin^2\theta}{g} + y_0 = y$

$-\frac{v_0^2 \sin^2\theta + 2v_0^2 \sin^2\theta}{2g} + y_0 = y$

$$\boxed{\frac{v_0^2 \sin^2\theta}{2g} + y_0 = y}$$ HEIGHT

NOW WE WANT TO FIND RANGE ($v_x = 0\,m/s$; $y = 0$)

FOR TIME

$a \quad b \quad c$

$-\frac{gt^2}{2} + v_{0y}t + y_0 = 0$

NOW WE USE THE QUADRATIC FORMULA

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(v_0 \sin\theta) \pm \sqrt{(v_0^2 \sin^2\theta) - 4(-g/2)(y_0)}}{2(-g/2)}$

ONLY + IS THE PHYSICALLY RIGHT SOLUTION

$$\boxed{t_{RANGE} = \frac{v_0 \sin\theta}{g} + \sqrt{\left(\frac{v_0 \sin\theta}{g}\right)^2 + \frac{2y_0}{g}}}$$

NOW PLUG IN t_{RANGE} INTO $x = v_{0x} t + x_0$

$$x = v_0 \cos\theta \left[\frac{v_0 \sin\theta}{g} + \sqrt{\left(\frac{v_0 \sin\theta}{g}\right)^2 + \frac{2y_0}{g}} \right] + x_0 \quad \text{RANGE}$$

AND THE FINAL VELOCITY

$$\vec{v}_f = v_0 \cos\theta \, \hat{i} + (v_0 \sin\theta - g t_{RANGE}) \hat{j}$$